B-Decay CP Asymmetries, Discrete Ambiguities and New Physics

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Abstract: The first measurements of CP violation in the B system will likely probe $\sin 2\alpha$, $\sin 2\beta$ and $\cos 2\gamma$. Assuming that the CP angles α, β and γ are the interior angles of the unitarity triangle, these measurements determine the angle set (α, β, γ) except for a twofold discrete ambiguity. If one allows for the possibility of new physics, the presence of this discrete ambiguity can make its discovery difficult: if only one of the two candidate solutions is consistent with constraints from other measurements in the B and K systems, one is not sure whether new physics is present or not. We review the methods used to resolve the discrete ambiguity and show that, even in the presence of new physics, they can usually be used to uncover this new physics. There are some exceptions, which we describe in detail. We systematically scan the parameter space and present examples of values of (α, β, γ) and the new-physics parameters which correspond to all possibilities. Finally, we show that if one relaxes the assumption that the bag parameters B_{B_d} and B_K are positive, one can no longer definitively establish the presence of new physics.

1   Introduction

In the Standard Model (SM), CP violation is due to nonzero complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix V. These CKM phases are elegantly described in terms of the interior angles α, β and γ of the "unitarity triangle" [1]. This triangle has two possible orientations. If we take the possible range of any angle to be $-\pi$ to $+\pi$, then, for one of these orientations, α, β and γ are all positive, while for the other they are all negative. Either way,

$$\alpha, \beta \text{ and } \gamma \text{ are all of the same sign.} \tag{1}$$

In addition, the angles in the unitarity triangle obviously satisfy the constraint

$$|\alpha + \beta + \gamma| = \pi . \tag{2}$$

The angles α, β and γ may be expressed in terms of the parameters ρ and η in Wolfenstein's approximation to the CKM matrix [2]. Existing information on $|V_{cb}|$, $|V_{ub}/V_{cb}|$, B_d and B_s mixing, and CP violation in the kaon system (ϵ_K) restricts ρ and η to the 95% confidence level allowed region shown in Fig. 1 [3]. Correspondingly, α, β and γ are restricted to the ranges

$$65° \leq \quad \alpha \quad \leq 123° , \tag{3}$$

$$16° \leq \quad \beta \quad \leq 35° , \tag{4}$$

$$37° \leq \quad \gamma \quad \leq 97° . \tag{5}$$

Note that the unitarity triangle shown in Fig. 1 points up, which implies that the CP angles α, β and γ are all positive. This is a consequence of the measured phase of ϵ_K and of the assumption that the kaon bag parameter B_K is positive [4, 5]. While lattice calculations firmly indicate that, in fact, $B_K > 0$, this has not been verified experimentally. If $B_K < 0$, the unitarity triangle shown in Fig. 1 points down, so that the CP angles are all negative, with the above allowed ranges changing sign as well. For the most part, in this paper we assume, as usual, that the lattice prediction that $B_K > 0$ is correct. However, we shall also include a separate discussion of the consequences for the search for new physics if it is not.

To test the SM picture of CP violation, and to look for evidence of new physics beyond the SM, coming experiments will attempt to determine the angles α, β and γ implied by CP-violating asymmetries in various B decays [6]. If the measured angles violate either of the "triangle conditions," Eqs. (1) and (2), or correspond to a point (ρ, η) which is outside the allowed region (Fig. 1), then we will have evidence of new physics.

The sign of the CP asymmetries in B_d decays is dependent on the sign of the bag parameter B_{B_d} [5]. Like B_K, B_{B_d} is firmly predicted by lattice calculations to be positive, and
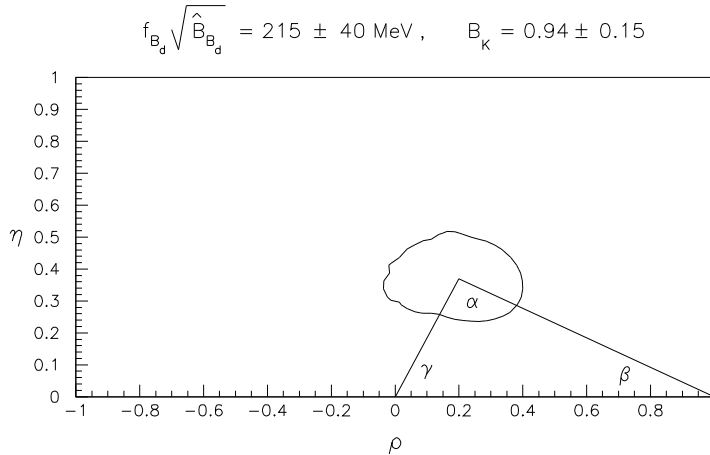
$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 215 \pm 40 \text{ MeV}, \qquad B_K = 0.94 \pm 0.15$$

Figure 1: Allowed region (95% C.L.) in ρ–η space in the SM [3].

is usually assumed to have this sign. Under this assumption, if measurements yield a unitarity triangle which points downward, i.e. one which is inconsistent with the measurement of ϵ_K, this implies that new physics is present, even if the angles of the downward-pointing triangle have magnitudes consistent with Eqs. (3)-(5). The CDF collaboration has recently reported that, at the 93% confidence level, $\Gamma[\overline{B_d}(t) \to \Psi K_s] > \Gamma[B_d(t) \to \Psi K_s]$ [7]. If we combine this result with the constraint of Eq. (4) on $|\beta|$, and assume that $B_{B_d} > 0$, this result implies that the unitarity triangle points upward, consistent with the implication of ϵ_K.

Unfortunately, the CP asymmetries in the B system do not directly determine the angles in the unitarity triangle. Rather, these asymmetries yield only trigonometric functions of these angles, such as $\sin 2\alpha$ and $\sin 2\beta$, leaving the underlying angles themselves discretely ambiguous. Needless to say, this makes the goal of testing for consistency with the SM and looking for evidence of physics beyond it more challenging.

The first CP asymmetry to be measured will almost certainly be the one in $B_d^0(t) \to \Psi K_s$. The second may well be the one in $B_d^0(t) \to \pi^+\pi^-$ (or $\pi^+\pi^-\pi^0$ [8]). Quite possibly, the third will be the one in $B^\pm \to DK^\pm$ [9]. We shall assume this scenario. Now, the CP asymmetry in any B decay probes the CP-odd part of the relative phase of (hopefully only) two interfering amplitudes. We shall call the CP-odd relative phases probed in $B_d^0(t) \to \pi^+\pi^-$, $B_d^0(t) \to \Psi K_s$ and $B^\pm \to DK^\pm$, respectively, $2\tilde{\alpha}$, $2\tilde{\beta}$ and $\tilde{\gamma}$.[1] The trigonometric functions of these phases which will be determined by the CP asymmetries in these three decays will be $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\sin^2 \tilde{\gamma}$ (or equivalently $\cos 2\tilde{\gamma}$), respectively.

As the notation suggests, when new physics is absent, $\tilde{\alpha} = \alpha$, $\tilde{\beta} = \beta$ and $\tilde{\gamma} = \gamma$.

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[1]In $B_d^0(t) \to \pi^+\pi^-$ there may be significant penguin contributions [10], so that there are more than two interfering amplitudes. If this is the case, we assume that an isospin analysis is employed to find the CP-odd relative phase in the absence of penguins [11].

3

Process	Relative Phase of Amplitudes	CP Asymmetry Measures	Value of Probed Phase
$B_d^0(t) \to \pi^+\pi^-$	$2\tilde{\alpha}$	$\sin 2\tilde{\alpha}$	$\tilde{\alpha} = \alpha + \theta_d$
$B_d^0(t) \to \Psi K_S$	$2\tilde{\beta}$	$\sin 2\tilde{\beta}$	$\tilde{\beta} = \beta - \theta_d$
$B^\pm \to DK^\pm$	$\tilde{\gamma}$	$\cos 2\tilde{\gamma}$	$\tilde{\gamma} = \gamma$

Table 1: The CP-violating phase information to be obtained from first-round B experiments on CP violation.

However, in the presence of new physics, the situation changes. Most likely, if new physics affects CP violation in the B system, it does so by changing the phase of the neutral B-$\bar{B}$ mixing amplitudes [12]. In this paper, we shall assume that new physics enters only in this way. In the presence of this new physics,

$$\arg A(B_q \to \overline{B_q}) = \arg A(B_q \to \overline{B_q})|_{SM} + 2\theta_q ; \qquad q = d, s . \qquad (6)$$

Here, $A(B_q \to \overline{B_q})$ is, of course, the $B_q \to \overline{B_q}$ amplitude, $\arg A(B_q \to \overline{B_q})|_{SM} = 2\arg(V_{tq}V_{tb}^*)$ is the phase of $B_q \to \overline{B_q}$ mixing in the Standard Model, and $2\theta_q$ is the change in this phase due to new physics. When this new physics is present, the phases probed by $B_d^0(t) \to \pi^+\pi^-$ and $B_d^0(t) \to \Psi K_S$ are changed to $\tilde{\alpha} = \alpha + \theta_d$ and $\tilde{\beta} = \beta - \theta_d$, respectively [13]. The phase $\tilde{\gamma}$ probed by $B^\pm \to DK^\pm$, which does not involve neutral B mixing, remains γ. The situation is summarized in Table 1.

To test the SM, it is reasonable to assume, at least provisionally, that no new physics is present, so that $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = (\alpha, \beta, \gamma)$. Then $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ satisfy the triangle conditions: $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are all of the same sign, and $|\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}| = \pi$. When this assumption is made, the quantities $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ to be measured by the early B CP experiments always leave a twofold discrete ambiguity in the angle-set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. As we shall see, at most one of these two candidate solutions is consistent with the SM. In order to establish the presence of new physics, it will be necessary to resolve the ambiguity between the two solutions. In this paper, we will refer to this as "discrete ambiguity resolution" (DAR).

Up to this point, the entire discussion has essentially been a review. It has been known for quite some time now that the presence of discrete ambiguities can make the discovery of new physics problematic, and various aspects of issues related to this problem have been discussed in the literature [4, 14, 15, 16, 17]. In Ref. [15], Wolfenstein presented a specific example of how a nonzero value of the new-physics phase θ_d may be difficult to detect due to discrete ambiguities. Grossman, Nir and Worah have proposed a method to measure the phase (and magnitude) of the new-physics contribution to B mixing [16]. However, their technique runs into serious difficulties due to discrete ambiguities. And Grossman and Quinn have discussed a variety of methods of removing discrete ambiguities [17].

What is missing from this collective analysis, however, is a complete, systematic study

of how the presence of new physics affects the resolution of the discrete ambiguities. For example, some of the DAR methods presented in Ref. [17] may not work if new physics is present. It is therefore important to re-examine the various DAR techniques, assuming the presence of new physics, to see exactly what information they give us. (After all, if one assumes that new physics is not present, then no DAR is necessary – one simply chooses the solution which is consistent with all other measurements.)

Furthermore, for arbitrary values of θ_d it is not always obvious just how DAR will work to reveal the presence of new physics. For example, as mentioned above, the measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ leave a twofold discrete ambiguity in the angle-set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. If there is no new physics ($\theta_d = 0$), then one angle-set will form a triangle which lies within the allowed region of Fig. 1 (the SM solution), while the other lies outside. DAR will then clearly choose the SM solution. If θ_d is relatively small, then the two candidate triangles will be close to the $\theta_d = 0$ triangles, and DAR will choose that triangle which is close to the SM solution. However, suppose now that θ_d is large. In this case, one or both of $\tilde{\alpha}$ and $\tilde{\beta}$ may change sign relative to the original α and β (see Table 1), so that the angle-set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ will not even form a triangle. Will DAR still reveal the presence of new physics in such a situation? And if it does, how are the angles of the triangle thus found related to the (non-triangle) angle-set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$? These types of questions must be investigated in order to understand how the presence of new physics may affect the resolution of discrete ambiguities.

Finally, one might wonder how "likely" it is that DAR will be necessary in order to discover new physics. That is, for what values of (α, β, γ) and θ_d will we find at least one solution which lies in the allowed region of Fig. 1?

In this paper we examine all of these issues, as well as some others. We begin in Sec. 2 with a review of the discrete ambiguities which are present when the quantities $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ are measured. (Note that other analyses [15, 16, 17] usually assume that only $\sin 2\tilde{\alpha}$ and $\sin 2\tilde{\beta}$ have been measured. However, since the measurement of the CP asymmetry in $B_d^0(t) \to \pi^+\pi^-$ is likely to be at least as difficult as that in $B^\pm \to DK^\pm$, it seems more reasonable to assume that both will be measured.) When $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ satisfy the triangle conditions of Eqs. (1) and (2), we show that these measurements always leave a twofold discrete ambiguity in the angle-set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. We will also show that at most one of these two candidate solutions is consistent with the SM.

We also briefly examine how the various DAR methods are affected by the presence of new physics. These methods can be separated into two categories: those that measure $\sin 2\phi$ or $\cos 2\phi$, where ϕ is some combination of CP phases, and those that measure $\sin \phi$. These latter methods are dependent on theoretical assumptions. As we shall see, in the presence of new physics, those methods which probe $\sin \phi$ may not correctly resolve the discrete ambiguity. However, those which measure $\sin 2\phi$ or $\cos 2\phi$ are still reliable.

In Sec. 3, we examine the question of how the presence of physics beyond the SM affects the search for this same new physics via the resolution of discrete ambiguities. Assuming that the new physics affects $\tilde{\alpha}$ and $\tilde{\beta}$ as indicated in Table 1, $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ still satisfy one of the triangle conditions: $|\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}| = \pi$. Thus, the presence of the new physics would not be revealed by looking for a violation of this condition. However, when θ_d is present, $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ may not all be of like sign, in violation of the other triangle condition, Eq. (1). If one could resolve the discrete ambiguities in $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ separately, without assuming that the phases satisfy the triangle conditions, then when $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are not of like sign one would immediately discover that new physics is present. Unfortunately, as indicated above, this is not possible: to fully resolve the discrete ambiguities in a CP phase ϕ, one needs $\sin\phi$, and the methods which probe $\sin\phi$ are not reliable in the presence of new physics.

Suppose, then, that one simply assumes provisionally that $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ do satisfy both triangle conditions, and in particular are of like sign. Under what circumstances would new physics still be uncovered? In Sec. 3 we shall see that whether or not θ_d results in $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ not being of like sign, if one assumes that they *are* of like sign, the measured quantities (Table 1, Column 3) always lead to two candidate solutions for $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. There are then three possibilities:

1. Both of the candidate $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ solutions obtained assuming $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are of like sign are consistent with the allowed (ρ, η) region in Fig. 1. As mentioned above, in Sec. 2, we will see that, in practice, this is impossible.

2. One of the candidate $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ solutions is consistent with the allowed (ρ, η) region, but the other is not.

3. Neither of the candidate $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ solutions is consistent with the allowed (ρ, η) region. In this case, it is clear that new physics is present.

Of these three possibilities, case (2) is obviously the one which causes problems. Even if physics beyond the SM is present, due to the existence of the twofold discrete ambiguity, the measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ alone will not unequivocally reveal its presence. In order to know whether or not new physics is present, it will be necessary to remove the discrete ambiguity.

When the true $(\tilde{\alpha}, \tilde{\beta}, \text{and } \tilde{\gamma})$ are of like sign, one of the two candidates for $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ is the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. In this situation, the discrete ambiguity resolution (DAR) techniques to be reviewed in Sec. 2 will select from among the two candidates the true one. When the true $(\tilde{\alpha}, \tilde{\beta}, \text{and } \tilde{\gamma})$ are not of like sign, neither of the candidates for $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ is the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$, and the DAR techniques simply select one or the other of the incorrect candidates. Suppose, now, that in the case (2) where one of the candidates is consistent

with the allowed (ρ, η) region but the other is not, DAR selects as the alleged true solution the one which is inconsistent with the allowed region. Then, regardless of the signs of the true $(\tilde{\alpha}, \tilde{\beta}, \text{ and } \tilde{\gamma})$, one would know for certain that new physics is present, and one would not have known this without the DAR. However, it might also happen that the DAR selects the candidate solution which is consistent with the allowed (ρ, η) region. Then it could be that no new physics is present and this solution represents the true angles α, β and γ in the unitarity triangle. But it could also be that new physics *is* present, but that the CP angles and θ_d are such that it remains hidden. In Sec. 3 we will provide illustrative examples of all of these situations.

The above analysis is done assuming that both B_K and B_{B_d} are positive. However, since these theoretical predictions have not been confirmed experimentally, it is conceivable that this assumption is incorrect. In Sec. 4 we examine the consequences for DAR and the search for new physics assuming that the signs of B_K and B_{B_d} are unknown. As we will show, under this assumption, DAR can *not* definitively establish the presence of new physics. We conclude in Sec. 5.

2 Discrete Ambiguities and Their Resolution

The CP asymmetries in the decays $B_d^0(t) \rightarrow \pi^+\pi^-$, $B_d^0(t) \rightarrow \Psi K_s$ and $B^\pm \rightarrow DK^\pm$ permit the extraction of the functions $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\sin^2 \tilde{\gamma}$ (or equivalently $\cos 2\tilde{\gamma}$), respectively[2]. (An alternative way of getting at $\tilde{\gamma}$ is through the CP asymmetry in $B_s^0(t) \rightarrow D_s^\pm K^\mp$ [18]. However, even in this case, the function measured is $\sin^2 \tilde{\gamma}$.) Thus, from these measurements, each CP angle can be obtained up to a fourfold ambiguity: if $\tilde{\alpha}_0$, $\tilde{\beta}_0$ and $\tilde{\gamma}_0$ are the true values of these angles, the values consistent with the measurements are:

$$
\begin{aligned}
&\tilde{\alpha}_0 \ , \ \tilde{\alpha}_0 + \pi \ , \ \frac{\pi}{2} - \tilde{\alpha}_0 \ , \ -\frac{\pi}{2} - \tilde{\alpha}_0 \ , \\
&\tilde{\beta}_0 \ , \ \tilde{\beta}_0 + \pi \ , \ \frac{\pi}{2} - \tilde{\beta}_0 \ , \ -\frac{\pi}{2} - \tilde{\beta}_0 \ , \\
&\tilde{\gamma}_0 \ , \ \tilde{\gamma}_0 + \pi \ , \ -\tilde{\gamma}_0 \ , \ -\tilde{\gamma}_0 - \pi \ .
\end{aligned}
\tag{7}
$$

There is thus a 64-fold discrete ambiguity in the extraction of the CP-angle set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. However, assuming that the three angles are the interior angles of a triangle, i.e. that they satisfy Eqs. (1) and (2), this discrete ambiguity can be reduced to a twofold one. This result can be found in the literature [17], although an explicit proof has not been given. For completeness, we present such a proof in this paper. However, since the main purpose

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[2]When an isospin analysis is used to extract $\sin 2\tilde{\alpha}$ from the decay $B_d^0(t) \rightarrow \pi^+\pi^-$ despite the presence of penguins, the net result is that $\sin 2\tilde{\alpha}$ is itself obtained with a fourfold discrete ambiguity, which depends on the relative magnitude and phase of the penguin and tree amplitudes. In this paper we ignore this ambiguity, since in general only one of the four values of $\sin 2\tilde{\alpha}$ yields values of $\tilde{\alpha}$ which can satisfy $|\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}| = \pi$. Furthermore, $\sin 2\tilde{\alpha}$ can be extracted independently with no discrete ambiguity from a study of $B \rightarrow \rho\pi$ decays [8].

$Sign(\sin 2\tilde{\alpha})$	$Sign(\sin 2\tilde{\beta})$	Discrete Ambiguity
> 0	> 0	$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right) \rightarrow \left(\frac{\pi}{2} - \tilde{\alpha}, \frac{\pi}{2} - \tilde{\beta}, \pi - \tilde{\gamma}\right)$
> 0	< 0	$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right) \rightarrow \left(-\frac{\pi}{2} - \tilde{\alpha}, \frac{\pi}{2} - \tilde{\beta}, -\tilde{\gamma}\right)$
< 0	> 0	$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right) \rightarrow \left(\frac{\pi}{2} - \tilde{\alpha}, -\frac{\pi}{2} - \tilde{\beta}, -\tilde{\gamma}\right)$
< 0	< 0	$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right) \rightarrow \left(-\frac{\pi}{2} - \tilde{\alpha}, -\frac{\pi}{2} - \tilde{\beta}, -\pi - \tilde{\gamma}\right)$

Table 2: The twofold discrete ambiguity in $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ remaining after measurement of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$.

of the paper is to examine the consequences of this discrete ambiguity, we defer the proof of its existence to Appendix A, and simply list the possible discrete ambiguities in Table 2.

There is one point which should be noted here. Within the SM, the magnitude of the angle β is constrained to be $16° \leq |\beta| \leq 35°$ [Eq. (3)]. However, an examination of Table 2 reveals that, regardless of $Sign(\sin 2\tilde{\beta})$, at most one of the two $\tilde{\beta}$ solutions satisfies this constraint (and it can be that neither does). Therefore, regardless of the signs of the candidate angle sets or of B_K and B_{B_d}, *at most one of the two discretely ambiguous solutions can be consistent with the SM*. This will have important consequences in our discussion of new physics.

If one assumes that there is no new physics (i.e. $\tilde{\alpha} = \alpha$, $\tilde{\beta} = \beta$, $\tilde{\gamma} = \gamma$), this twofold discrete ambiguity does not pose a problem. Since only one of the two solutions can be consistent with the SM, then clearly that is the one which must be chosen.

On the other hand, if one allows for the possibility that new physics may affect the CP asymmetries, then there may be a problem. If both solutions are inconsistent with the SM, then it is clear that new physics is present. However, if one solution is consistent with the SM, while the other is not, then the measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ alone will not tell us which of the two solutions is the correct one. In other words, at this stage we will not know whether or not new physics is present. In order to decide, it will be important to be able to remove the discrete ambiguities. Below, we briefly review various ways of doing this.

2.1   Discrete Ambiguity Resolution (1)

The methods for discrete ambiguity resolution can be separated into two categories. First, techniques involving indirect, mixing-induced CP violation can be used to extract trigonometric functions of the CP phases other than $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$. For example, $\cos 2\tilde{\alpha}$ can be obtained through a study of the time-dependent Dalitz plot for $B_d^0(t) \rightarrow \rho\pi$ decays [8]. Similarly, Dalitz-plot analyses of the decays $B_d^0(t) \rightarrow D^+D^-K_s$ and $B_d^0(t) \rightarrow D^{\pm}\pi^{\mp}K_s$

allow one to extract the functions $\cos 2\tilde{\beta}$ and $\sin 2(2\tilde{\beta} + \tilde{\gamma})$, respectively [19]. $\cos 2\tilde{\beta}$ can also be obtained through a study of $B_d^0 \to \Psi + K \to \Psi + (\pi^- \ell^+ \nu)$, known as "cascade mixing" [20, 21]. Finally, $\sin 2\tilde{\gamma}$ can be obtained from $B_s^0(t) \to D_s^{\pm} K^{\mp}$ if the width difference between the two B_s mass eigenstates is measurable [22]. For a more complete discussion of these techniques, we refer to the reader to Refs. [17] and [23].

Knowledge of any of these functions — $\cos 2\tilde{\alpha}$, $\cos 2\tilde{\beta}$, $\sin 2(2\tilde{\beta} + \tilde{\gamma})$, or $\sin 2\tilde{\gamma}$ — is sufficient to remove the discrete ambiguity left by the measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$. (There are also several CP asymmetries that can be used to extract $\sin^2(2\tilde{\beta} + \tilde{\gamma})$: $B_d^0(t) \to D^{(**)} K_s$, $B_d^0(t) \to D^{\mp} \pi^{\pm}$ [24]. However, it is straightforward to show that the discrete ambiguities in Table 2 are *not* resolved by this measurement.) In fact, if $\sin 2\tilde{\gamma}$ is known, the measurement of $\cos 2\tilde{\gamma}$ is not even necessary. Assuming that $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ obey the triangle conditions [Eqs. (1) and (2)], except for certain singular values of the CP angles, the measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\sin 2\tilde{\gamma}$ determine $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ uniquely. This fact was also pointed out in Ref. [4], though the proof was not given. We present the proof in Appendix B.

Actually, the above analysis is not complete. If there is no new physics, i.e. $\theta_d = 0$, then $\tilde{\alpha} = \alpha$, $\tilde{\beta} = \beta$, and $\tilde{\gamma} = \gamma$. In this case it is true that, as described above, each of the various methods does probe the given trigonometric function of the CP angles, and the discrete ambiguity is indeed resolved by any of the above measurements. However, as we have argued earlier, this is not particularly relevant – if no new physics is assumed, there is no need for DAR. What we really want to know is the following: in the presence of new physics, do the above methods still probe the same functions of the CP angles as they did in the absence of new physics? That is, are we certain that we have taken into account all possible large new-physics effects?

In fact, for this class of DAR methods, the answer to these questions is yes. Even in the presence of new physics, each technique probes the same function of CP angles as it did in the absence of new physics, with (obviously) α being replaced by $\tilde{\alpha}$, and similarly for β and γ. We demonstrate this explicitly for cascade mixing.

The decay $B_d^0 \to \Psi + (\pi^- \ell^+ \nu)_K$ can occur via the path $B_d^0 \to \Psi + K^0 \to \Psi + (\pi^- \ell^+ \nu)_K$ and the path $B_d^0 \to \overline{B_d^0} \to \Psi + \overline{K^0} \to \Psi + K^0 \to \Psi + (\pi^- \ell^+ \nu)_K$. In the latter path, neutral K mixing follows neutral B mixing. In the absence of new physics, the interference between the two paths probes $\sin 2\beta$ and $\cos 2\beta$. Now, new physics can modify both the phase and the magnitude of the $B_d^0 \to \overline{B_d^0}$ mixing amplitude. Since, in the presence of new physics, $\arg A(B_d \to \overline{B_d}) = -2(\beta - \theta_d) = -2\tilde{\beta}$, the first of these effects is taken into account by replacing β by $\tilde{\beta}$ in the expression for the decay rate. The second is taken into account by using the physical neutral B mass splitting. New physics can conceivably also affect $K^0 \to \overline{K^0}$ mixing, but this is taken into account by using the physical neutral K mass splitting and decay widths in the expression for the rate. The CP-violating phase

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of $K^0 \to \overline{K^0}$ mixing is known to be small, whether it involves new physics or not. In particular, it is negligible compared to the effects expected in the B system. Finally, significant new-physics effects on the $B \to \Psi K$ decay amplitudes are not expected. Thus, the replacement of β by $\tilde{\beta}$ and the use of the observed mass splittings and widths takes into account all significant new-physics effects.

Although we will not show it explicitly, it is not difficult to convince oneself that a similar logic holds for the other DAR methods which rely on indirect CP violation. In all cases, the only place where there can be significant new-physics effects is in B mixing, and this is taken into account by the replacement of α, β and γ by $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$, respectively. We therefore conclude that these methods can be reliably used to remove the discrete ambiguity even in the presence of new physics.

In the illustrative case of cascade mixing, one finds by explicit calculation that [21]

$$
\begin{aligned}
\Gamma \left[B_d^0 \to \Psi + K \to \Psi + (\pi^- \ell^+ \nu) \right] & \\
\propto e^{-\Gamma_B \tau_B} \Big\{ & e^{-\gamma_S \tau_K} \left[1 - \sin 2\tilde{\beta} \, \sin(\Delta M_B \tau_B) \right] \\
& + e^{-\gamma_L \tau_K} \left[1 + \sin 2\tilde{\beta} \, \sin(\Delta M_B \tau_B) \right] \\
& + 2 \, e^{-\frac{1}{2}(\gamma_S + \gamma_L)\tau_K} \left[\cos(\Delta M_B \tau_B) \, \cos(\Delta M_K \tau_K) \right. \\
& \left. + \cos 2\tilde{\beta} \, \sin(\Delta M_B \tau_B) \, \sin(\Delta M_K \tau_K) \right] \Big\} \, .
\end{aligned}
\tag{8}
$$

Here, Γ_B is the width of the B, γ_S and γ_L are, respectively, the widths of K_S and K_L, ΔM_B and ΔM_K are the positive B and K mass splittings, and τ_B and τ_K are the proper times that the B and K live before decay. A similar expression holds when the initial state is a $\overline{B_d^0}$, or the final state is $\Psi + (\pi^+ \ell^- \bar{\nu})$.

Note that, although the expression of the rate for $B_d^0 \to \Psi + (\pi^- \ell^+ \nu)$ is more complicated than that for $B_d^0 \to \Psi K_S$, it is still independent of hadronic uncertainties. It depends only on the CP angle $\tilde{\beta}$, and on the known quantities Γ_B, γ_S, γ_L, ΔM_B and ΔM_K. The key point is that the function $\cos 2\tilde{\beta}$ appears in the expression for the rate. Thus, this method allows one to measure $\cos 2\tilde{\beta}$ and thus remove the discrete ambiguity of Table 2.

2.2 Discrete Ambiguity Resolution (2)

The second category of DAR methods are those which use direct CP violation, and probe the sine of some CP phase [17]. For example, recall that CP violation in $B_d^0(t) \to \Psi K_S$ probes $\sin 2\tilde{\beta}$. In principle, the decay $B_d^0(t) \to D^+ D^-$ can also be used to extract $\sin 2\tilde{\beta}$. However, this decay may not be dominated by a single decay amplitude — there may be a significant penguin contribution — so the CP-violation measurement will not be free of hadronic uncertainties. Still, by comparing the two measurements and using some theoretical input, one can obtain $\cos 2\tilde{\beta} \sin \tilde{\beta}$, or at least its sign. Similarly a comparison

of CP violation in $B_d^0(t) \to \rho\pi$ and $B_d^0(t) \to \pi^+\pi^- 0$ gives information about $\cos 2\tilde{\alpha} \sin \tilde{\alpha}$. Any such information may help in reducing the discrete ambiguities.

Unfortunately, there are problems with such methods. Even leaving aside the question of the theoretical input, if there is new physics, these methods cannot be reliably used for discrete ambiguity resolution. The reason is that new physics can affect not only B mixing, but also the penguin contributions to B decays [25]. As before, the easiest way to see this is with an example.

The decay $B_d^0 \to D^+D^-$ receives contributions from a tree amplitude and a $b \to d$ penguin amplitude, which itself has contributions from internal u, c and t-quarks. Eliminating the u-quark penguin piece via CKM unitarity, and combining the c-quark contribution with the tree contribution, in the Wolfenstein approximation the total amplitude for $B_d^0 \to D^+D^-$ can be written

$$
\begin{aligned}
A &= A_T e^{i\delta_T} + A_P e^{i\delta_P} e^{-i\beta'} \ , \\
\bar{A} &= A_T e^{i\delta_T} + A_P e^{i\delta_P} e^{i\beta'} \ .
\end{aligned}
\tag{9}
$$

Here we have written the weak phase of the t-quark penguin contribution as β'. In the SM, this phase is β. However, it is conceivable that new physics can contribute to the $b \to d$ penguin, thereby altering this phase.

By measuring the time-dependent rate for $B_d^0(t) \to D^+D^-$, one can extract the CP asymmetry $a_{D^+D^-}^{\sin}$, defined as

$$
a_{D^+D^-}^{\sin} \equiv \frac{-2 \, \mathrm{Im}\lambda}{1 + |\lambda|^2} \ ,
\tag{10}
$$

where

$$
\lambda \equiv \frac{q}{p} \frac{\bar{A}}{A} \ .
\tag{11}
$$

Here q/p is the phase of B mixing. Allowing for the presence of new physics in the mixing, this can be written as

$$
\frac{q}{p} = e^{-2i\tilde{\beta}} \ .
\tag{12}
$$

For the final state D^+D^-, the CP asymmetry is

$$
a_{D^+D^-}^{\sin} \simeq \sin 2\tilde{\beta} - 2r \cos \delta \cos 2\tilde{\beta} \sin \beta' \ ,
\tag{13}
$$

where $\delta \equiv \delta_P - \delta_T$, $r \equiv A_P/A_T$, and we have neglected terms of $O(r^2)$. The function $\sin 2\tilde{\beta}$ is assumed to have been measured in $B_d^0(t) \to \Psi K_S$ and it is also assumed that we have some reliable theoretical input concerning the hadronic factor $r \cos \delta$. Now, in the absence of new physics, $\tilde{\beta} = \beta' = \beta$. Thus, the measurement of $a_{D^+D^-}^{\sin}$ would allow us to obtain information about $\cos 2\beta \sin \beta$. However, this no longer holds in the presence of new physics. Any new physics which contributes to B_d^0-$\overline{B_d^0}$ mixing will, in general, also contribute to the $b \to d$ penguin. However, it will not necessarily contribute in the same

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way. That is, there is no reason to expect that $\tilde{\beta} = \beta'$. In particular, it is conceivable that the new physics is such that $\tilde{\beta}$ and β' are of opposite sign, in which case even knowledge of the sign of $\cos 2\tilde{\beta} \sin \beta'$ may not tell us anything about the sign of $\tilde{\beta}$. We therefore conclude that this method, and others like it, cannot be reliably used to remove discrete ambiguities if there is new physics. (Similar conclusions have been reached in Refs. [17] and [15].)

2.3   Reduction of the Allowed (ρ, η) Region

Finally, we reiterate that the main problem caused by the presence of the twofold discrete ambiguity is the possibility of having one solution inside the allowed region of Fig. 1, and the other outside. In this case, one does not know whether or not new physics is present. Above we reviewed the measurements which can be used to remove the discrete ambiguity. However, there is another approach which can be used. If the allowed region of Fig. 1 were reduced, this would then reduce the likelihood of having one solution inside the allowed region in the presence of new physics.

The measurements which contribute to the allowed region of Fig. 1 are $|V_{cb}|$, $|V_{ub}/V_{cb}|$, B_d and B_s mixing, and CP violation in the kaon system (ϵ_K). If the error on any of these measurements can be reduced, either through reduced experimental error or better understanding of the theoretical uncertainties, this would help to reduce the allowed region. In fact, over the past year or two, the improved lower bound on B_s mixing has already removed roughly half of the previously-allowed (ρ, η) region. An actual value for this mixing would be helpful indeed.

There is another measurement which can help to reduce the allowed (ρ, η) region. Within the SM, the decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ probes the Wolfenstein parameter η. In the Wolfenstein parametrization, the branching ratio can be written [26]:

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 3.0 \times 10^{-11} \left[\frac{\eta}{0.39} \right]^2 \left[\frac{\bar{m}_t(m_t)}{170 \text{ GeV}} \right]^{2.3} \left[\frac{|V_{cb}|}{0.040} \right]^4 . \tag{14}$$

Since the branching ratio is proportional to η^2, its measurement would greatly help in constraining the allowed (ρ, η) region.

Note that the precision with which η can be extracted from a measurement of $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is mainly limited by the error on $|V_{cb}|$, which presently stands at about 4.3% [1], including both the experimental and theoretical (HQET) errors. If the error on $|V_{cb}|$ could be reduced, then this measurement could be used to obtain $|\eta|$, the height of the unitarity triangle.

3 Finding New Physics

Let us now turn to the question of what specifically is learned when discrete ambiguity resolution is applied. As usual, we assume that CP violation in the B system is affected by new physics (NP) beyond the Standard Model. In particular, we assume that the NP affects CP-violating asymmetries by modifying the phase of B_d^0-$\overline{B_d^0}$ mixing, as described in the Introduction [Eq. (6)]. The phases probed by the first-round CP experiments on the B system are then the quantities $\tilde{\alpha} = \alpha + \theta_d$, $\tilde{\beta} = \beta - \theta_d$ and $\tilde{\gamma} = \gamma$ given in the last column of Table 1.

As we have noted, while $|\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}| = \pi$, in the presence of a nonzero θ_d, $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ may not all be of the same sign, unlike the true angles α, β and γ in the unitarity triangle. Resolving the discrete ambiguities in $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ separately, and finding that they are not all of like sign, would reveal the presence of NP. However, as already mentioned, we do not have a method for reliably resolving the ambiguities in $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ separately. Therefore, we ask under what conditions the NP would still be visible even if one proceeds by simply *assuming* that $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ satisfy both of the "triangle conditions" [Eqs. (1) and (2)].

To answer this question, we first show that, given measured values of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$, there are always two candidate solutions for $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ which satisfy both triangle conditions. We call such solutions "triangle angle sets." To demonstrate that two of them always exist, we distinguish two possibilities:

1. Suppose that, even with a nonzero θ_d, $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are all of the same sign (that is, positive). Then the angle set $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ clearly satisfies the two triangle conditions of Eqs. (1) and (2). There is then a second triangle angle set with the same values of the measured quantities. It is related to $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ by one of the entries in Table 2. Note that one of these two triangle angle sets is obviously the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$.

2. Suppose, instead, that θ_d is such that $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are not of the same sign. This can occur in a number of ways. For example, θ_d can be such that $\tilde{\alpha} = \alpha + \theta_d$ satisfies $-\pi < \tilde{\alpha} < 0$, while $\tilde{\beta} = \beta - \theta_d$ satisfies $0 < \tilde{\beta} < \pi$. Or perhaps the result is $0 < \tilde{\alpha} < \pi$ and $-\pi < \tilde{\beta} < 0$. We refer to these situations, where θ_d has flipped the sign of α or β, but not both, as a "single flip." (Of course, $\tilde{\gamma} = \gamma$ is unaffected by θ_d, and so remains positive.) It is also possible that θ_d flips the sign of both α and β – a "double flip." In all cases, one ends up with two of the three CP angles being of like sign, with the third having the opposite sign.

   In all of these cases, to obtain a triangle angle set, one has simply to add $\pm\pi$ to the two angles which have the same sign. This will give three same-sign angles which form a triangle and reproduce the measured quantities. And, as above, the second triangle angle set is obtained from the appropriate entry in Table 2. However, in

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Angle(s) flipped	$(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$
none	$\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right)$
α	$\left(\tilde{\alpha}, \tilde{\beta} - \pi, \tilde{\gamma} - \pi\right)$
β	$\left(\tilde{\alpha} - \pi, \tilde{\beta}, \tilde{\gamma} - \pi\right)$
α, β (1)	$\left(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma}\right)$
α, β (2)	$\left(\tilde{\alpha} + \pi, \tilde{\beta} - \pi, \tilde{\gamma}\right)$

Table 3: Construction of the triangle angle set $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ in the presence of NP. The second triangle angle set $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ can be obtained from the appropriate entry in Table 2. (1) refers to the case where $\pi < \tilde{\alpha} < 2\pi$ and $-\pi < \tilde{\beta} < 0$; (2) refers to $-\pi < \tilde{\alpha} < 0$ and $\pi < \tilde{\beta} < 2\pi$.

contrast to the previous case, neither of these two candidate triangle angle sets is the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$.

In Table 3 we summarize the relation between the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ and the triangle set $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ most simply related to it.

In summary, if NP enters by modifying B_d^0-$\overline{B_d^0}$ mixing, then there are always two triangle angle sets, $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ and $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$, which give *any* observed values of the measured quantities. These two sets are related by one of the entries in Table 2, depending on the signs of $\sin 2\tilde{\alpha}$ and $\sin 2\tilde{\beta}$. If the true $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are of like sign (no flips), then one of these two angle sets, which we call $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$, is the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. If the true $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are not of like sign (one or two flips), then obviously neither $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ nor $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is the true $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$.

Suppose, now, that $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ are measured. Suppose further that one then proceeds by assuming the underlying angles $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ to be the angles α, β and γ in the unitarity triangle, and looking for inconsistencies. One is then assuming that $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ is a triangle angle set, so one would calculate that $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ is either the set of angles we have called $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$, or the one we have called $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$. As already stated in the Introduction, one will then encounter one of the following three situations:

1. Both $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ and $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ are consistent with the allowed (ρ, η) region in Fig. 1.

2. Only one of $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ and $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is consistent with the allowed (ρ, η) region.

3. Neither $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ nor $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is consistent with the allowed (ρ, η) region.

As we have already argued in Sec. 2, in practice situation (1) can never occur. Only one of the two solutions related as in Table 2 will be consistent with known physics. However,

if new physics is indeed present, then resolution of the discrete ambiguity will in fact establish its presence, except in some specific cases which we describe below.

We are assuming that new physics affects CP violation in the B system only by changing the phase of neutral B mixing. We have seen that when this is the case, there are always two candidate triangle angle sets consistent with any given values of the measured quantities $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$. Thus, if no other quantities are measured, one cannot uncover the presence of the NP by trying to determine whether the angles underlying the measured $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ form a triangle angle set. Can one, alternatively, uncover it by determining that the candidate triangle angle sets correspond to points (ρ, η) which are outside the allowed region? A study of Table 3 shows that the answer to this question is usually yes.

First, we note that, in Table 3, $\tilde{\alpha}_1$ is always equal to $\tilde{\alpha}$, $\tilde{\alpha} + \pi$ or $\tilde{\alpha} - \pi$, and similarly for $\tilde{\beta}_1$ and $\tilde{\gamma}_1$. The second angle set $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is obtained from the appropriate entry in Table 2. This is important when one considers the resolution of the twofold discrete ambiguity. In Sec. 2, we reviewed various measurements which can be reliably used for DAR. In all cases, one of the following trigonometric functions is obtained: $\cos 2\tilde{\alpha}$, $\cos 2\tilde{\beta}$, $\sin 2\tilde{\gamma}$ or $\sin 2(2\tilde{\beta} + \tilde{\gamma})$. The key point is that all of these functions are unchanged when one replaces $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ by $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$. On the other hand, these functions change sign if $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is used. In other words, *in all cases, the resolution of the discrete ambiguity chooses the angle set* $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$.

From this we conclude that discrete ambiguity resolution will always reveal the presence of new physics unless $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ corresponds to a triangle which falls within the allowed region of Fig. 1. And, by studying Table 3, it is straightforward to see when this occurs. Whenever there is one flip (i.e. the second and third lines of Table 3), one ends up with a downward-pointing unitarity triangle, which is always a signal of new physics (recall that we are assuming that B_K is positive). If there are no flips (i.e. the first line of Table 3), the resultant triangle will lie within the allowed region if $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) \simeq (\alpha, \beta, \gamma)$. This can only happen if θ_d is small. Similarly, if there are two flips (i.e. the fourth and fifth lines of Table 3), the resultant triangle will be consistent with other measurements only if $\theta_d \pm \pi$ is small. We therefore conclude that DAR will always reveal the presence of new physics unless the new-physics angle θ_d is near 0 or π. (And note: since only the quantity $2\theta_d$ ever enters into CP asymmetries [see Eq. (6)], a new-physics phase $\theta_d \simeq \pi$ is essentially equivalent to $\theta_d \simeq 0$. In other words, in both cases, we are effectively talking about small new-physics effects.)

Of course, this raises the following question: what precisely does "near 0" mean? The easiest way to answer this is to refer to Eq. 4, which gives the currently-allowed range for β as $16° \leq \beta \leq 35°$. Depending on the true value of β, for the case of $\theta_d \simeq 0$, θ_d must be small enough that the value of $\tilde{\beta} = \beta - \theta_d$ still lies within this range. In such

a situation the new-physics triangle will still lie within the allowed region of Fig. 1 and DAR will not discover the presence of new physics. Similarly, if $\theta_d \simeq \pi$, $\tilde{\beta}$ must lie within the range $-164° \leq \tilde{\beta} \leq -145°$. In this way $\tilde{\beta} + \pi$ will fall in the allowed range for β, and again DAR will not reveal that new physics is present. This shows explicitly that a reduction of the allowed (ρ, η) region, which would reduce the allowed range for β, would help considerably in the search for new physics.

Once the discrete ambiguities have been removed, the method of Grossman, Nir and Worah (GNW) [16] can in principle be used to measure the new-physics phase θ_d. Among other things, this method assumes that we know the new-physics triangle, defined by the angles $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. However, as we have noted above, even when DAR does reveal the presence of new physics, it does not always choose the true values of $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. In such cases, the GNW method will not find the true value of θ_d, but rather $\theta_d + \pi$. Still, as noted above (and in Ref. [16]), as far as B mixing is concerned, this difference is not physical.

Above we have seen that there is uncertainty about the presence of new physics only if one of the two discretely ambiguous solutions is consistent with the allowed (ρ, η) region, while the other is not. And we have seen that DAR is needed to remove this uncertainty. But this also suggests that, for other values of θ_d, both solutions will lie outside the allowed (ρ, η) region. In such cases, we will know that new physics is present, even without DAR. The obvious question then is: how likely is it that DAR will be needed?

While we cannot answer this question in any statistically rigorous way, one can still get a sense of how things work by performing a scan of the (α, β, γ) and θ_d parameter space. The results of such an analysis are shown in Tables 4, 5, 6, 7 and 8, which correspond, respectively, to the assumption that the true angles in the unitarity triangle correspond to a point (ρ, η) which is near the center of the (ρ, η) region allowed by Fig. 1, near the left edge of this region, right edge, the top, and the bottom[3]. For each of these five sample cases, we explore the effect of a θ_d of 22°, 45°, 78° and 90°. We give for each θ_d the two candidate triangle angle sets, $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ and $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$, that would be inferred from the measured $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$. Next to each angle set, we give the corresponding inferred (ρ, η) point.

Each of Tables 4-8 covers values of $2\theta_d$, the angle which actually enters in $A(B_d^0 \to \overline{B_d^0})$ [see Eq. (6)], spanning the full range $(0, \pi)$. The effect of a negative $2\theta_d$ in the range $(-\pi, 0)$ can be deduced from that of the positive angle $2\theta_d + \pi$, since $2\theta_d$ leads to values of $\sin 2\tilde{\alpha}$ and $\sin 2\tilde{\beta}$ opposite to those produced by $2\theta_d + \pi$. Thus, the candidate triangle angle sets

---

[3]In all of our examples, we assume that the true (α, β, γ) correspond to a (ρ, η) within or very near the allowed (ρ, η) region of Fig. 1. However, it should be noted that since the new physics affects B mixing, which is one of the inputs to Fig. 1, that region may not represent the true allowed (ρ, η) region. In other words, in the presence of new physics, the true SM (α, β, γ) may already lie outside the region of Fig. 1. In this paper, we do not consider this additional possibility, but its inclusion would not affect our conclusions.

θ_d	$\tilde{\alpha}_1,\ \tilde{\beta}_1,\ \tilde{\gamma}_1$	(ρ, η)	$\tilde{\alpha}_2,\ \tilde{\beta}_2,\ \tilde{\gamma}_2$	(ρ, η)
22°	117°, 3°, 60°	(0.03, 0.05)	−27°, −93°, −60°	(1.10, −1.90)
45°	−40°, −20°, −120°	(−0.27, −0.46)	−50°, −70°, −60°	(0.61, −1.06)
68°	−17°, −43°, −120°	(−1.17, −2.02)	−73°, −47°, −60°	(0.38, −0.66)
90°	5°, 115°, 60°	(5.20, 9.01)	−95°, −25°, −60°	(0.21, −0.37)

Table 4: The effects of new physics when the true angles in the unitarity triangle are $(\alpha, \beta, \gamma) = (95°, 25°, 60°)$, corresponding to $(\rho, \eta) = (0.21, 0.37)$.

θ_d	$\tilde{\alpha}_1,\ \tilde{\beta}_1,\ \tilde{\gamma}_1$	(ρ, η)	$\tilde{\alpha}_2,\ \tilde{\beta}_2,\ \tilde{\gamma}_2$	(ρ, η)
22°	−83°, −2°, −95°	(0.0, −0.04)	−7°, −88°, −85°	(0.71, −8.17)
45°	−60°, −25°, −95°	(−0.04, −0.49)	−30°, −65°, −85°	(0.16, −1.81)
68°	−37°, −48°, −95°	(−0.11, −1.23)	−53°, −42°, −85°	(0.07, −0.83)
90°	−15°, −70°, −95°	(−0.32, −3.62)	−75°, −20°, −85°	(0.03, −0.35)

Table 5: Same as Table 4, but with the true $(\alpha, \beta, \gamma) = (75°, 20°, 85°)$, corresponding to $(\rho, \eta) = (0.03, 0.35)$.

θ_d	$\tilde{\alpha}_1,\ \tilde{\beta}_1,\ \tilde{\gamma}_1$	(ρ, η)	$\tilde{\alpha}_2,\ \tilde{\beta}_2,\ \tilde{\gamma}_2$	(ρ, η)
22°	132°, 8°, 40°	(0.14, 0.12)	−42°, −98°, −40°	(1.13, −0.95)
45°	−25°, −15°, −140°	(−0.47, −0.39)	−65°, −75°, −40°	(0.82, −0.69)
68°	−2°, −38°, −140°	(−13.51, −11.34)	−88°, −52°, −40°	(0.60, −0.51)
90°	20°, 120°, 40°	(1.94, 1.63)	−110°, −30°, −40°	(0.41, −0.34)

Table 6: Same as Table 4, but with the true $(\alpha, \beta, \gamma) = (110°, 30°, 40°)$, corresponding to $(\rho, \eta) = (0.41, 0.34)$.

θ_d	$\tilde{\alpha}_1,\ \tilde{\beta}_1,\ \tilde{\gamma}_1$	(ρ, η)	$\tilde{\alpha}_2,\ \tilde{\beta}_2,\ \tilde{\gamma}_2$	(ρ, η)
22°	107°, 8°, 65°	(0.06, 0.13)	−17°, −98°, −65°	(1.43, −3.07)
45°	−50°, −15°, −115°	(−0.14, −0.31)	−40°, −75°, −65°	(0.64, −1.36)
68°	−27°, −38°, −115°	(−0.57, −1.23)	−63°, −52°, −65°	(0.37, −0.80)
90°	−5°, −60°, −115°	(−4.20, −9.01)	−85°, −30°, −65°	(0.21, −0.45)

Table 7: Same as Table 4, but with the true $(\alpha, \beta, \gamma) = (85°, 30°, 65°)$, corresponding to $(\rho, \eta) = (0.21, 0.45)$.

θ_d	$\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1$	(ρ, η)	$\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2$	(ρ, η)
$22°$	$-45°, -5°, -130°$	$(-0.08, -0.09)$	$-45°, -85°, -50°$	$(0.91, -1.08)$
$45°$	$-22°, -28°, -130°$	$(-0.81, -0.96)$	$-68°, -62°, -50°$	$(0.61, -0.73)$
$68°$	$1°, 129°, 50°$	$(28.62, 34.11)$	$-91°, -39°, -50°$	$(0.40, -0.48)$
$90°$	$23°, 107°, 50°$	$(1.57, 1.87)$	$-113°, -17°, -50°$	$(0.20, -0.24)$

Table 8: Same as Table 4, but with the true $(\alpha, \beta, \gamma) = (113°, 17°, 50°)$, corresponding to $(\rho, \eta) = (0.20, 0.24)$.

$(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$ and $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ which correspond to a NP angle $2\theta_d$ are just the negatives of those for $2\theta_d + \pi$.

Perusal of Tables 4-8 reveals several interesting points. First, we note that most of the candidate angle sets in the survey represented by these Tables correspond to (ρ, η) points well outside both the allowed (ρ, η) region in Fig. 1 and its $\eta \to -\eta$ mirror image. Whenever both candidate angle sets for a given true (α, β, γ) and θ_d have (ρ, η) values outside the allowed region, accurate measurements of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ would make the presence of new physics clear. We notice, however, that some of the candidate angle sets in Tables 4-8 have (ρ, η) values rather close to, or actually inside, the mirror image of the allowed region. This means that the candidates obtained when $2\theta_d$ is replaced by $2\theta_d - \pi$ would have (ρ, η) values inside the allowed region itself. Whenever this happens, one would not know new physics is present without measuring additional quantities.

As Tables 4-8 illustrate, if $\theta_d = \frac{\pi}{2}$, the candidate angle set $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ is always identical to the true (α, β, γ), except that the unitarity triangle has been flipped over (the common sign of the angles has been reversed, and η has been replaced by $-\eta$). The reason for this is simply that when $\theta_d = \pm\frac{\pi}{2}$, $\sin 2\tilde{\alpha} = -\sin 2\alpha$ and $\sin 2\tilde{\beta} = -\sin 2\beta$. Thus, since $\cos 2\tilde{\gamma}$ is insensitive to the sign of $\tilde{\gamma}$, one of the candidate triangles looks like the real unitarity triangle, but flipped. If one assumes that the theoretical signs of B_{B_d} and B_K are correct, the flipped character of this triangle would imply that it cannot be the true unitarity triangle. Thus, if, as in all of the $\theta_d = \frac{\pi}{2}$ examples in Tables 4-8, the other candidate triangle, $(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1)$, is also inconsistent with the allowed (ρ, η) region, one would conclude that new physics is present. This conclusion would be confirmed by resolving the discrete ambiguity, since as we have seen this resolution would always reject the triangle $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$, which here is the candidate mirroring the true unitarity triangle.

As Tables 4-8 also show, values of θ_d other than $\frac{\pi}{2}$ can also lead to candidate triangles which are at least close to being consistent with the allowed (ρ, η) region of Fig. 1 or its $\eta \to -\eta$ mirror image. One interesting example of this phenomenon is the second row of Table 5, for the case $(\alpha, \beta, \gamma) = (75°, 20°, 85°)$ and $\theta_d = 45°$. From this entry, it follows

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that $\theta_d = -45°$ would lead to the candidate triangles

$$(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (30°, 65°, 85°) \tag{15}$$

with

$$(\rho, \eta) = (0.16, 1.81) \ , \tag{16}$$

and

$$(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (60°, 25°, 95°) \tag{17}$$

with

$$(\rho, \eta) = (-0.04, 0.49) \ . \tag{18}$$

This first of these is clearly inconsistent with the allowed (ρ, η) region of Fig. 1, but the second is rather close to being consistent with it. Absent any additional information, one would not know whether new physics is present or not. However, as always, DAR would rule out candidate 2 – the triangle which is close to consistency with the Standard Model – leaving only candidate 1, which is completely inconsistent with the SM. Once again, new physics would thereby be clearly established.

In fact, by changing the parameters of this example slightly, we can easily produce a case where, despite a large θ_d, one of the candidate triangles has a (ρ, η) actually inside the allowed region. Suppose that $(\alpha, \beta, \gamma) = (70°, 20°, 90°)$, so that $(\rho, \eta) = (0.0, 0.36)$ is inside the allowed region, and that $\theta_d = -50°$. The measured $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\cos 2\tilde{\gamma}$ would then lead to the candidate triangles

$$(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (20°, 70°, 90°) \tag{19}$$

with

$$(\rho, \eta) = (0.00, 2.75) \ , \tag{20}$$

and

$$(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (70°, 20°, 90°) \tag{21}$$

with

$$(\rho, \eta) = (0.00, 0.36) \ . \tag{22}$$

Triangle 1 is completely inconsistent with the allowed (ρ, η) region, but triangle 2 happens to coincide with the true SM unitarity triangle (despite the presence of a large θ_d!), and so is totally consistent with the SM. As above, a DAR would select candidate 1, which is inconsistent with the allowed (ρ, η) region. Thus, once again, such a measurement would establish that new physics is present.

As these examples explicitly illustrate, if one of the two candidate triangles is fully consistent with the allowed (ρ, η) region and the other is far from consistent with it, the assumption that the consistent candidate represents the true CP-violating phases in B

decay and no new physics is present can be completely erroneous. In these examples the resolution of the discrete ambiguity would reveal that the Standard Model-consistent candidate does *not* represent the true CP phases, and that new physics *is* present.

As a final example, consider the third line of Table 8. From this entry, it follows that if θ_d were $-22°$, the two candidate triangles would be

$$(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (91°, 39°, 50°) \tag{23}$$

with

$$(\rho, \eta) = (0.40, 0.48) , \tag{24}$$

and

$$(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (-1°, -129°, -50°) \tag{25}$$

with

$$(\rho, \eta) = (28.62, -34.11) . \tag{26}$$

Here, triangle 1 is (nearly) consistent with the allowed (ρ, η) region, while triangle 2 is not. In this situation, DAR, by ruling out triangle 2 as always, will select the consistent solution. This might tempt one to erroneously conclude that there is no new physics present. However, as this example illustrates, large new-physics effects may actually be present in CP asymmetries even when these asymmetries appear to be consistent with the SM. Although this appears to contradict the conclusions of the earlier discussion, in fact this is an example of a "small" θ_d. That is, the true value of β is $17°$, near the lower bound of Eq. 4, while the new-physics value is $\tilde{\beta}_1 = 39°$, very near the upper bound. In other words, the new physics has essentially moved the apex of the unitarity triangle from a point within the allowed (ρ, η) region to another point in the region. In such cases, DAR will not help to uncover the new physics. As noted before, this is why a reduction of the allowed region would be quite helpful.

To summarize: from the expressions which determine the candidate triangles 1 and 2 that correspond to given (α, β, γ) and θ_d, it is relatively straightforward to show that neither candidate will be consistent with the presently-allowed (ρ, η) region unless

$$(i) \quad 80° \lesssim \gamma \lesssim 100° \tag{27}$$

or

$$(ii) \quad 0 \le |\theta_d| \lesssim 20° . \tag{28}$$

However, when one of these conditions is met, or approximately met, then as the examples we have considered show, it is indeed possible for a candidate triangle to be nearly or fully consistent with the allowed region. When this occurs, one cannot establish that new physics is present without an additional measurement. Resolution of the discrete ambiguity, as described in Sec. 2, would often provide the needed information. The only exception is when θ_d is close to 0 or π.

4 B_{B_d} and B_K

It is interesting to notice that the conclusion that new physics is present can sometimes depend crucially on the theoretical signs of the bag parameters B_{B_d} and B_K being correct. To see this, suppose that one of the two candidate triangles implied by the measured values of $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$, and $\cos 2\tilde{\gamma}$ is consistent with the allowed (ρ, η) region or with its $\eta \to -\eta$ mirror image, while the other candidate triangle is not. Suppose further that a DAR selects the triangle which is not consistent with either the allowed (ρ, η) region or its mirror image. If we assume that the theoretical signs of B_{B_d} and B_K are correct, as we have been doing, then we can conclude that new physics is present. Imagine, however, that we allow for the possibility that the theoretical $\text{Sign}(B_{B_d})$ and/or $\text{Sign}(B_K)$ is wrong. Can we still conclude that new physics is present?

The answer to this question is "no". The main point is that our assumed techniques for determining $\sin 2\tilde{\alpha}$ and $\sin 2\tilde{\beta}$, and for DAR, all depend on an interference between a B_d^0 decay-path which involves $B_d^0 - \overline{B_d^0}$ mixing and one which does not. If the $\text{Sign}(B_{B_d})$ used when interpreting the data is wrong, then the assumed relation between the measured interference and the underlying parameters will be wrong. One can show that, as a result, the extracted $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$, and any of the quantities (such as $\cos 2\tilde{\alpha}$) to be used by DAR, will have the wrong sign. This means that the two candidate triangles deduced from the data will be upside down relative to what they should be, and that the DAR will select the wrong candidate triangle. Thus, when DAR appears to select the candidate triangle which is not consistent with the allowed (ρ, η) region, this could be due to $\text{Sign}(B_{B_d})$ being wrong, rather than to this triangle representing the truth and new physics being present. The ability to conclude unambiguously that new physics is present has been lost.[4]

The situation is summarized in detail in Table 9. In constructing this table, we have assumed that one of the two candidate triangles is consistent with the allowed (ρ, η) region or with its mirror image, that the other candidate triangle is not consistent with either, and that DAR selects the latter triangle. For all possible orientations of the candidate triangles, we indicate whether these circumstances still imply the presence of new physics when the theoretical sign of B_K and/or B_{B_d} is wrong. Table 9 makes clear that, regardless of the orientations of the candidate triangles, if the theoretical $\text{Sign}(B_{B_d})$ might be wrong, then one cannot unambiguously conclude that new physics is present. To be sure, if the allowed-(ρ, η)-region-consistent candidate triangle points up, then this conclusion *is* still possible if one knows for certain that the theoretical $\text{Sign}(B_K)$ is correct.

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[4]This is also true when one measures $\sin 2\tilde{\gamma}$ in $B_s^0(t) \to D_s^{\pm} K^{\mp}$, which involves $B_s^0 - \overline{B_s^0}$, rather than $B_d^0 - \overline{B_d^0}$, mixing. As we have noted, when $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$, and $\sin 2\tilde{\gamma}$ are known, there is only one candidate triangle. However, if the signs of the bag parameters used to determine this triangle are wrong, it may not represent the true $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$. Suppose, for instance, that $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = (70°, 30°, 80°)$. If $\text{Sign}(B_{B_d})$ is right, but $\text{Sign}(B_{B_s})$ is wrong, we will get $(20°, 60°, 100°)$.

Allowed (ρ, η) Consistent Candidate	Allowed (ρ, η) Inconsistent Candidate	B_K	B_{B_d}	NP Definitely Present
Up	Up	Right	Right	Yes
		Wrong	Right	Yes
		Right	Wrong	Yes
		Wrong	Wrong	No
Down	Down	Right	Right	Yes
		Wrong	Right	Yes
		Right	Wrong	No
		Wrong	Wrong	Yes
Up	Down	Right	Right	Yes
		Wrong	Right	Yes
		Right	Wrong	Yes
		Wrong	Wrong	No
Down	Up	Right	Right	Yes
		Wrong	Right	Yes
		Right	Wrong	No
		Wrong	Wrong	Yes

Table 9: The effect of incorrect signs of B_K and B_{B_d} on one's ability to conclude that new physics (NP) is present. In the first column, we indicate whether the candidate triangle consistent with the allowed (ρ, η) region or its mirror image points up $(\eta > 0)$ or down $(\eta < 0)$. In the second column, we show the same thing for the candidate triangle which is not consistent with either the allowed (ρ, η) region or its mirror image. In the third and fourth columns, we indicate whether the theoretical signs of B_K and B_{B_d} are right or wrong. In the final column, we indicate whether, under the stated assumptions concerning the signs of B_K and B_{B_d}, one would still be able to conclude that NP is definitely present.

However, realistically, if one is unsure of $\mathrm{Sign}(B_{B_d})$, then one is unsure of $\mathrm{Sign}(B_K)$ as well.

None of this is meant to cast doubt on the theoretically-determined signs of B_K and B_{B_d}. However, since these signs are not experimentally verified, it is important to recognize the crucial role that they may prove to play in establishing the existence of new physics.

5   Conclusions

Within the standard model, CP violation is due to complex phases in the CKM matrix. This explanation will be tested through the measurement of CP-violating asymmetries in the B system. Such measurements will permit the extraction of the interior angles α, β and γ of the unitarity triangle. If the unitarity triangle constructed from these CP angles is inconsistent with the (ρ, η) region allowed by other measurements ($|V_{cb}|$, $|V_{ub}/V_{cb}|$, B_d

and B_s mixing, ϵ_K), this will signal the presence of new physics.

Unfortunately, it is not the CP angles themselves which will be measured, but rather trigonometric functions of these angles. In particular, it is very likely that the first measurements will extract the functions $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2 \gamma$ (or equivalently $\cos 2\gamma$). This implies that the CP angles can be obtained only up to discrete ambiguities. In this paper we have demonstrated that, under the assumption that the angles are the interior angles of a triangle (i.e. that they are all of the same sign and add up to $\pm\pi$), a twofold discrete ambiguity remains in the triangle angle set (α, β, γ). That is, there are two sets of solutions which form a triangle and still reproduce the experimental results.

If one does not allow for the possibility of new physics, this discrete ambiguity causes no problems. As shown in the paper, at most one of these two solutions is consistent with present experimental constraints on the unitarity triangle. Thus, in the absence of new physics, one simply chooses that solution which is consistent with the allowed (ρ, η) region.

On the other hand, if one allows for the possibility of new physics, then there may be a problem. In the presence of new physics which modifies the phase of $B_d^0 - \overline{B_d^0}$ mixing, the CP angles measured are not the SM angles α, β and γ, but rather $\tilde{\alpha} = \alpha + \theta_d$, $\tilde{\beta} = \beta - \theta_d$ and $\tilde{\gamma} = \gamma$, where θ_d is the modification due to new physics. Even in this case, though, there are still two candidate triangle angle sets. If both solutions are inconsistent with the allowed (ρ, η) region, then new physics is clearly present. But if one solution is consistent with this region, while the other is not, then one cannot be certain whether new physics is or is not present. In this case it is necessary to resolve the discrete ambiguity.

We have briefly reviewed the various methods for discrete ambiguity resolution (DAR). There is a class of techniques which use measurements of indirect, mixing-induced CP violation to extract different trigonometric functions of the CP angles: $\cos 2\tilde{\alpha}$, $\cos 2\tilde{\beta}$, $\sin 2\tilde{\gamma}$, or $\sin 2(2\tilde{\beta} + \tilde{\gamma})$. Any one of these measurements is sufficent to remove the twofold discrete ambiguity, even in the presence of new physics. (In fact, if one measures $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$ and $\sin 2\tilde{\gamma}$, there is no discrete ambiguity at all in the triangle angle set – the measurement of $\cos 2\tilde{\gamma}$ is not even necessary.) There is also a second class of techniques which rely on direct CP violation due to the interference of a tree and a penguin amplitude. However, these cannot be used to reliably resolve the discrete ambiguity if new physics is present. The reason is that new physics can affect not only B mixing, but also the penguin contributions, and these new-physics effects may be quite different.

In this paper, we have systematically studied what kind of information is obtained from the resolution of the discrete ambiguity when new physics is present. We have shown that, regardless of the value of θ_d, DAR almost always reveals the presence of new physics by choosing the solution which is inconsistent with the allowed (ρ, η) region. The only exception is when θ_d is near 0 or π. In addition, we have shown that, even when

DAR does show that new physics is present, it does not always choose the true values of $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$.

We have explicitly demonstrated all of these points with the help of several examples. By scanning over the parameter space, we have found a number of different values of the SM (α, β, γ) and new-physics θ_d which yield the situation in which one triangle angle set is consistent with the allowed (ρ, η) region, while the other is not. In most of these examples, the DAR chooses the solution which is inconsistent with the allowed (ρ, η) region, thereby demonstrating that new physics is present. Without DAR, one might have been led (erroneously) to think that the solution which is consistent with the allowed region is the true SM solution, and no new physics is present. The only examples in which DAR fails are those for which θ_d is small.

Note that the analysis of our examples is strongly dependent on the size of the allowed (ρ, η) region. Any reduction in the allowed region, such as an actual measurement of B_s mixing or the measurement of $B(K_L \to \pi^0 \nu \bar{\nu})$, would reduce the likelihood that one of the two candidate triangles will be consistent with the region even when new physics is present. (Indeed, the increasingly-stringent lower limits on B_s mixing have already helped in this regard.) If the region were sufficiently reduced, then, except for some very fine-tuned choices of θ_d, DAR would not be necessary at all.

Finally, the above analysis is based on the assumption that the theoretical signs of B_{B_d} and B_K are correct. We have shown that if we relax this assumption, DAR will fail. That is, if we allow B_{B_d} and B_K to be either positive or negative, then DAR cannot definitively establish the presence of new physics. It is therefore important to try to verify experimentally the signs of these quantities.

Appendix A

In this Appendix, we present the proof that, assuming that the three angles α, β and γ are the interior angles of a triangle, the functions $\sin 2\alpha$, $\sin 2\beta$ and $\cos 2\gamma$ determine the angle set (α, β, γ) up to a twofold ambiguity.

First, suppose that $\sin 2\alpha$ and $\sin 2\beta$ have the same sign, e.g. assume that they are both positive. Since α and β must have the same sign [see Eq. (1)], this implies that 2α and 2β both take values in the domain $(-2\pi, -\pi)$ or $(0, \pi)$. We can immediately exclude the $(-2\pi, -\pi)$ domain: since $|2\alpha| > \pi$ and $|2\beta| > \pi$, this implies that $|\alpha| + |\beta| > \pi$, in violation of Eq. (2). Thus, for $\phi = \alpha, \beta$ we can write

$$2\phi = \frac{\pi}{2} + 2\delta_\phi \quad , \qquad |\delta_\phi| < \frac{\pi}{4} \ . \tag{29}$$

The magnitude of δ_ϕ, *but not its sign*, is fixed by the measured value of $\sin 2\phi$. From Eq. (29) and the assumption that $|\alpha + \beta + \gamma| = \pi$, we have

$$\begin{aligned}
\alpha &= \frac{\pi}{4} + \delta_\alpha \ , \\
\beta &= \frac{\pi}{4} + \delta_\beta \ , \\
\gamma &= \frac{\pi}{2} - (\delta_\alpha + \delta_\beta) \ .
\end{aligned} \tag{30}$$

Now, the measured value of $\cos 2\gamma = -\cos 2(\delta_\alpha + \delta_\beta)$ gives us the relative sign of δ_α and δ_β. However, there still remains a twofold sign ambiguity, corresponding to $\delta_{\alpha,\beta} \to -\delta_{\alpha,\beta}$. This is equivalent to the twofold discrete ambiguity

$$(\alpha, \beta, \gamma) \to \left(\frac{\pi}{2} - \alpha, \frac{\pi}{2} - \beta, \pi - \gamma \right). \tag{31}$$

Note that both sets of CP angles in the above discrete ambiguity correspond to unitarity triangles which point up.

A similar analysis holds when $\sin 2\alpha$ and $\sin 2\beta$ are both negative, except that in this case the discrete ambiguity is between two downward-pointing unitarity triangles:

$$(\alpha, \beta, \gamma) \to \left(-\frac{\pi}{2} - \alpha, -\frac{\pi}{2} - \beta, -\pi - \gamma \right). \tag{32}$$

Now suppose that $\sin 2\alpha$ is positive and $\sin 2\beta$ is negative. Thus, 2α lies in the domain $(-2\pi, -\pi)$ or $(0, \pi)$, while 2β is in $(-\pi, 0)$ or $(\pi, 2\pi)$. There are now two cases to consider:

1. If α, β, γ are all positive, then

$$\begin{aligned}
2\alpha &= \frac{\pi}{2} + 2\delta_\alpha \ , \tag{33} \\
2\beta &= \frac{3\pi}{2} + 2\delta_\beta \ , \tag{34}
\end{aligned}$$

where $|\delta_\alpha|, |\delta_\beta| < \pi/4$. The measured values of $\sin 2\alpha$ and $\sin 2\beta$ determine the magnitudes of δ_α and δ_β, but not their signs. From Eq. (34) and the assumption that $|\alpha + \beta + \gamma| = \pi$, we have

$$\begin{aligned}
\alpha &= \frac{\pi}{4} + \delta_\alpha \ , \\
\beta &= \frac{3\pi}{4} + \delta_\beta \ , \\
\gamma &= -(\delta_\alpha + \delta_\beta) \ .
\end{aligned} \tag{35}$$

The requirement that $\gamma > 0$ implies that $(\delta_\alpha + \delta_\beta) < 0$. Now, the measured value of $\cos 2\gamma = \cos 2(\delta_\alpha + \delta_\beta)$ gives us the relative sign of δ_α and δ_β. This, along with the constraint that $(\delta_\alpha + \delta_\beta) < 0$, fixes δ_α and δ_β uniquely.

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2. If α, β, γ are all negative, then

$$
\begin{aligned}
2\alpha &= -\frac{3\pi}{2} - 2\delta_\alpha \ , \\
2\beta &= -\frac{\pi}{2} - 2\delta_\beta \ ,
\end{aligned}
\tag{36}
$$

with $|\delta_\alpha|, |\delta_\beta| < \pi/4$. Again, the measured values of $\sin 2\alpha$ and $\sin 2\beta$ determine the magnitudes of $\delta_{\alpha,\beta}$, but not their signs. We have

$$
\begin{aligned}
\alpha &= -\frac{3\pi}{4} - \delta_\alpha \ , \\
\beta &= -\frac{\pi}{4} - \delta_\beta \ , \\
\gamma &= \delta_\alpha + \delta_\beta \ .
\end{aligned}
\tag{37}
$$

This time, since $\gamma < 0$, one again requires that $(\delta_\alpha + \delta_\beta) < 0$. As before, the measured value of $\cos 2\gamma = \cos 2(\delta_\alpha + \delta_\beta)$ gives us the relative sign of δ_α and δ_β. And again, this fixes δ_α and δ_β uniquely once one takes into account the constraint that $(\delta_\alpha + \delta_\beta) < 0$.

Thus, for the case of $\sin 2\alpha > 0$ and $\sin 2\beta < 0$, we have two possible solutions for (α, β, γ): one with positive values, and the other with negative values. Now, from the definitions of δ_α and δ_β, it is clear that the magnitudes of these quantities are the same in both cases and are determined by the measured values of $\sin 2\alpha$ and $\sin 2\beta$. Furthermore, for both solutions we have $(\delta_\alpha + \delta_\beta) < 0$, with the relative sign being determined by the measurement of $\cos 2\gamma$. Thus, the two solutions have the *same* values of δ_α and δ_β. This allows us to determine the discrete ambiguity in this case. Denoting (α, β, γ) as the positive-angle solution above, the discrete ambiguity is

$$
(\alpha, \beta, \gamma) \rightarrow \left(-\frac{\pi}{2} - \alpha, \frac{\pi}{2} - \beta, -\gamma \right).
\tag{38}
$$

In this case, the first solution corresponds to a unitarity triangle pointing up, while the second corresponds to one pointing down.

Finally, the analysis in the case in which $\sin 2\alpha < 0$ and $\sin 2\beta > 0$ is clear from the above: the roles of α and β are reversed, and we have the following discrete ambiguity:

$$
(\alpha, \beta, \gamma) \rightarrow \left(\frac{\pi}{2} - \alpha, -\frac{\pi}{2} - \beta, -\gamma \right).
\tag{39}
$$

Appendix B

In this Appendix, we present the proof that, assuming that the three angles α, β and γ are the interior angles of a triangle, the functions $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ determine the angle set (α, β, γ) uniquely.

If $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ are all measured, then either (i) all three quantities are of the same sign, or (ii) two of the three quantities are of the same sign. We consider these two possibilities in turn.

Suppose first that $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ are all positive. As in Appendix A, this implies that $0 < \alpha, \beta, \gamma < \pi/2$. We can thus write

$$
\begin{aligned}
\alpha &= \frac{\pi}{4} + \delta_\alpha \ , \\
\beta &= \frac{\pi}{4} + \delta_\beta \ , \\
\gamma &= \frac{\pi}{2} - (\delta_\alpha + \delta_\beta) \ ,
\end{aligned}
\tag{40}
$$

where $\Delta_i \equiv |\delta_i| < \pi/4$, $i = \alpha, \beta$. Since $\gamma < \pi/2$, the bigger of δ_α and δ_β must be positive. Suppose that it is δ_α. Then, depending on the sign of δ_β, the quantity $\delta_\alpha + \delta_\beta$ is equal to either $\Delta_\alpha + \Delta_\beta$ or $\Delta_\alpha - \Delta_\beta$. However, in general the measured value of $\sin 2\gamma = \sin 2(\delta_\alpha + \delta_\beta)$ will distinguish between these two possibilities, and so measurements of $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ will determine α, β and γ uniquely. The one exception is the singular point $\Delta_\alpha = \pi/4$, or $\alpha = \pi/2$ ($\sin 2\alpha = 0$). In this case a twofold discrete ambiguity remains. A similar analysis holds if $\Delta_\beta > \Delta_\alpha$, and also when $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ are all negative.

Now suppose that two of these quantities are positive, and the third negative, say $\sin 2\alpha$, $\sin 2\beta > 0$ and $\sin 2\gamma < 0$. This implies that $0 < \alpha, \beta < \pi/2$ and $\pi/2 < \gamma < \pi$. Thus, we have again

$$
\begin{aligned}
\alpha &= \frac{\pi}{4} + \delta_\alpha \ , \\
\beta &= \frac{\pi}{4} + \delta_\beta \ , \\
\gamma &= \frac{\pi}{2} - (\delta_\alpha + \delta_\beta) \ ,
\end{aligned}
\tag{41}
$$

but this time the bigger of δ_α and δ_β must be negative. Again, suppose that it is δ_α, so that $\delta_\alpha + \delta_\beta$ is equal to either $-(\Delta_\alpha + \Delta_\beta)$ or $-(\Delta_\alpha - \Delta_\beta)$. In general the measured value of $\sin 2\gamma = \sin 2(\delta_\alpha + \delta_\beta)$ will distinguish between these two possibilities, so that measurements of $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ will determine α, β and γ uniquely. This will fail only in the special case in which $\Delta_\alpha = \pi/4$, or $\alpha = \pi/2$ ($\sin 2\alpha = 0$). The case $\Delta_\beta > \Delta_\alpha$ can be treated in the same way. A similar analysis holds when any two of $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ are of one sign, with the third of opposite sign.

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